



Warsaw Center
of Mathematics
and Computer Science



BANACH
CENTER

miniPAGES Polish Algebraic GEometry mini-Semester 18 April - 18 June 2016

Varieties with trivial canonical bundles

12-18 June 2016, Będlewo (near Poznań)

Program

Monday, June 13th

- 08.00-09.00 breakfast
09.00-09.30 registration
09.30-10.30 **S. Mukai:** *Root systems and automorphism groups of Enriques surfaces*
10.30-11.00 coffeebreak
11.00-12.00 **K. Ranestad:** *Kummer surfaces and hyperkähler fourfolds, as degeneracy loci*
12.15-13.00 lunch
14.30-15.30 **E. Amerik:** *Automorphisms of hyperkaehler manifolds via lattice embeddings*
15.30-16.00 coffeebreak
16.00-17.00 **F. Charles:** *Birational boundedness for K3 surfaces and arithmetic applications*
17.15-18.00 **A. Weber:** *Hirzebruch class of quotient varieties*
18.30 dinner

Tuesday, June 14th

- 08.00-09.00 breakfast
09.30-10.30 **JH. Keum:** *Curves of low degree on fake projective planes*
10.30-11.00 coffeebreak
11.00-12.00 **M. Schütt:** *Gorenstein \mathbb{Q}_1 -cohomology fake projective planes*
12.15-13.00 lunch
14.30-15.30 **T. Peternell:** *Nef line bundles on Calabi-Yau varieties*
15.30-16.00 coffeebreak
16.00-17.00 **V. Lazić:** *The existence of morphisms from a Calabi-Yau variety*
17.15-18.00 **S. Tanimoto:** *On the geometry of thin exceptional sets in Manin's conjecture*
18.30 conference dinner

Wednesday, June 15th

- 08.00-09.00 breakfast
09.00-10.00 **V. Nikulin:** *Degenerations of Kahlerian K3 surfaces with finite symplectic automorphism groups*
10.00-10.30 coffeekbreak
10.30-11.30 **A. Sarti:** *On the moduli space of smooth cubic threefolds and irreducible holomorphic symplectic manifolds*
12.00-12.45 lunch
13.00 excursion
20.00 dinner

Thursday, June 16th

- 08.00-09.00 breakfast
09.30-10.30 **E. Markman:** *Constructing algebraic cycles on products of K3 surfaces via hyperholomorphic bundles*
10.30-11.00 coffeekbreak
11.00-12.00 **L. Kamenova:** *Finiteness results for deformation types of hyperkähler manifolds*
12.15-13.00 lunch
14.30-15.30 **M. Brion:** *Minimal rational curves on almost homogeneous varieties*
15.30-16.00 coffeekbreak
16.00-16.40 **H. Castejón Díaz:** *Berezin–Toeplitz Quantization on K3 Surfaces*
16.45-17.25 **M. Dawes:** *Orthogonal modular varieties and the moduli of deformation generalised Kummer varieties*
17.30-18.15 **C. Di Natale:** *Hodge Theory and Deformations of Affine Cones of Sub-canonical Projective Varieties*
18.30 barbecue

Friday, June 17th

- 08.00-09.00 breakfast
09.30-10.30 **K. Hulek:** *On the (intersection) cohomology of \mathcal{A}_g and its compactifications*
10.30-11.00 coffeekbreak
11.00-12.00 **J-M. Hwang:** *Prolongation of infinitesimal automorphisms of projective varieties*
12.15-13.00 lunch
14.30-15.10 **S. Kapfer:** *Integral cohomology of the generalized Kummer fourfold*
15.15-15.55 **T.L. Kelly:** *Derived Equivalences of (Stacky) Calabi-Yau Hypersurfaces in Toric Varieties*
16.00-16.30 coffeekbreak
16.30-17.10 **A. Krug:** *Varieties with P -units*
17.15-18.00 **D. Festi:** *The Picard lattice of a family of double covers of \mathbb{P}^2*
18.30 dinner

Abstracts

Ekaterina Amerik

Automorphisms of hyperkaehler manifolds via lattice embeddings

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This is a joint work with Misha Verbitsky. In an earlier paper we have shown that the Beauville-Bogomolov squares of classes bounding the Kaehler cone on an irreducible holomorphic symplectic manifold are bounded in absolute value (a version of the Morrison-Kawamata cone conjecture). A recent refinement of this result states that the bound in question only depends on the topology of the manifold. This allows us to show that e.g. every irreducible holomorphic symplectic manifold with $b_2 \geq 5$ has a deformation admitting a positive entropy automorphism, by studying the primitive sublattices in the second cohomology.

Michel Brion

Minimal rational curves on almost homogeneous varieties

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A family of rational curves on a smooth projective variety is called minimal if the subfamily of curves through a general point is proper. Minimal rational curves and the associated varieties of minimal rational tangents feature prominently in the geometry of uniruled varieties. The latter include almost homogeneous varieties, that is, varieties on which a linear algebraic group acts with an open orbit. The talk (based on joint work with Fu Baohua) will present results and questions about minimal rational curves on these varieties, with special emphasis on equivariant compactifications of reductive groups.

Hèctor Castejón Díaz

Berezin–Toeplitz Quantization on K3 Surfaces

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A *Quantizable* manifold is a Kähler manifold (M, g, I, ω) together with a very ample line bundle $L \rightarrow M$ such that $\text{curv}(L) = -i\omega$, called *(pre)quantum Line Bundle*. The *Berezin–Toeplitz Quantization* of M is an operator which assigns canonically to each smooth function on M an operator acting on the Hilbert space $H^0(M, L)$ of holomorphic sections of L . Consider a K3 surface with its three Kähler structures (g, I_i, ω_{I_i}) , $i = 1, 2, 3$.

Assuming that all three structures are quantizable, one wonders if there exists any relation between the Quantum Line Bundles and their Hilbert spaces of holomorphic sections. During my talk I will explain the basic properties of quantization and I will show some relations between the different Quantum Hilbert spaces. I will also define a new generalization of the Berezin–Toeplitz operators for the tensor product of the different Quantum Hilbert spaces

$$\bigotimes_{i=1}^3 H^0(M, L_i)$$

and I will show that those new operators inherit the most important properties of the original ones.

Francois Charles

Birational boundedness for K3 surfaces and arithmetic applications

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We will investigate some boundedness results for line bundles on holomorphic symplectic varieties and give applications to algebraic cycles over number fields.

Matthew Dawes

Title: Orthogonal modular varieties and the moduli of deformation generalised Kummer varieties

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Moduli spaces of polarised irreducible symplectic manifolds are related to quotients of Hermitian symmetric domains of type IV by discrete subgroups of the orthogonal group $O(2, n)$. These quotients are known as ‘orthogonal modular varieties’ and many are expected to be of general type. Such results are typically proved by constructing pluricanonical forms on a smooth projective model of the modular variety from a supply of modular forms for the orthogonal group. The modular forms must be chosen to satisfy certain conditions that are imposed by the geometry of the modular variety.

In recent years, through the work of Gritsenko, Hulek, and Sankaran, much has been learned about this problem for orthogonal modular varieties of large dimension (such as those associated with the moduli of K3 surfaces, or with the moduli of irreducible symplectic manifolds of deformation $K3^{[n]}$ -type); less is known about modular varieties of small dimension, such as those associated with the moduli of deformation generalised Kummer varieties.

In particular, for modular varieties of small dimension, one needs a more detailed understanding of the singularities that can occur.

I shall survey this problem and discuss some results about the geometry of certain orthogonal modular varieties associated with the moduli of deformation generalised Kummer varieties.

Carmelo Di Natale

Hodge Theory and Deformations of Affine Cones of Subcanonical Projective Varieties

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This is a joint work with E. Fatighenti (Warwick) and D. Fiorenza (Roma). We investigate the relation between the Hodge theory of a smooth subcanonical n -dimensional projective variety X and the deformation theory of the affine cone A_X over X . We start by identifying $H_{\text{prim}}^{n-1,1}(X)$ as a distinguished graded component of the module of first order deformations of A_X , and later on we show how to identify the whole primitive cohomology of X as a distinguished graded component of the Hochschild cohomology module of the punctured affine cone over X . In the particular case of a projective smooth hypersurface X we recover Griffiths’ isomorphism between the primitive cohomology of X and certain distinguished graded components of the Milnor algebra of a polynomial defining X . The main result of the article can be effectively exploited to compute Hodge numbers of smooth subcanonical projective varieties.

Dino Festi

The Picard lattice of a family of double covers of \mathbb{P}^2

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A K3 surface is a smooth projective surface with trivial canonical divisor and trivial first cohomology group. A double cover of \mathbb{P}^2 ramified along a smooth sextic curve is an example of K3 surface. To every K3 surface it is possible to associate a lattice, called Picard (or Néron–Severi) lattice. The Picard lattice encodes important information about the arithmetic and the geometry of the surface, and it is often not easy to compute. In this talk we will consider a 1-dimensional family of K3 surfaces that are double covers of \mathbb{P}^2 ramified along an almost diagonal sextic, and we will see how we can compute the Picard lattice of the generic member of the family.

This is a joint work with Florian Bouyer, Edgar Costa, Chris Nicholls, and McKenzie West.

Klaus Hulek

On the (intersection) cohomology of \mathcal{A}_g and its compactifications

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In this talk we shall discuss cohomology and intersection cohomology of \mathcal{A}_g and its compactifications. This question has two aspects: namely the stabilization of (intersection) cohomology on the one hand and the computation of the entire cohomology for small values of g , on the other hand. I shall discuss both of these aspects. The emphasis of this talk will lie on recent computations of the intersection cohomology of the Satake compactification of $\mathcal{A}_g^{\text{Sat}}$ for small g (joint work with S. Grushevsky).

Jun-Muk Hwang

Prolongation of infinitesimal automorphisms of projective varieties

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We discuss the prolongations of the Lie algebras of infinitesimal automorphisms of projective varieties. The main problem is to classify varieties for which the prolongations are nonzero. This problem has been settled for nondegenerate nonsingular varieties in a joint work with Baohua Fu. We will discuss some new related results and applications.

Ljudmila Kamenova

Finiteness results for deformation types of hyperkähler manifolds

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In this lecture we will begin with a review of D. Huybrechts's classical finiteness results and we shall discuss F. Charles's boundedness results for holomorphic symplectic varieties. Then we'll prove finiteness for deformation types of hyperkähler manifolds in any fixed dimension with fixed Fujiki constant and discriminant of the Beauville-Bogomolov-Fujiki lattice. We will also prove finiteness of deformation types of hyperkähler Lagrangian fibrations with an ample line bundle of a given degree on the general fiber of the fibration.

Simon Kapfer

Integral cohomology of the generalized Kummer fourfold

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Rational cohomology of Hilbert schemes of points on surfaces is well understood in terms of Nakajima operators. If the surface is K3, one can use them to describe integral cohomology, too. Moreover, if the surface is a complex torus, it gives a partial description of integral cohomology of the generalized Kummer varieties, and in the case of the Kummer fourfold this picture can be completed.

Tyler L. Kelly

Derived Equivalences of (Stacky) Calabi-Yau Hypersurfaces in Toric Varieties

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Given Calabi-Yau hypersurfaces in a fixed toric variety, there are various constructions to find its mirror. Sometimes they are isomorphic, but sometimes they are not. Mirror symmetry predicts they still should be equivalent in some sense. In this talk, we will show that these (stacky) mirrors are birational and derived equivalent. If we have time, we will describe applications to more general contexts, depending on audience interest, about either lattice polarisations of families of K3 surfaces in toric varieties or extensions to Calabi-Yau complete intersections in toric varieties.

JongHae Keum

Curves of low degree on fake projective planes

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Fake projective planes are smooth projective surfaces with $p_g = q = 0$ and $K^2 = 9$. They are ball quotients. I will explain some fake projective planes do not contain curves of small degree, and then discuss some interesting consequences (of the non-existence of such curves), such as on the bicanonical map and the existence of exceptional collections.

Andreas Krug

Varieties with P -units

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This talk is about the class of compact Kähler manifolds with trivial canonical bundle such that the cohomology of the trivial line bundle is generated by one element. If the square of the generator is zero, we get the class of strict Calabi–Yau manifolds. If the generator is of degree 2, we get the class of compact hyperkähler manifolds. We provide some examples and structure results for the cases where the generator is of higher nilpotency index and degree. It turns out that varieties of this type are closely related to higher-dimensional Enriques varieties.

Vladimir Lazić

The existence of morphisms from a Calabi-Yau variety

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Given an algebraic variety, finding non-trivial morphisms to other varieties is one of the fundamental problems in algebraic geometry. For varieties with trivial canonical class, this can be formulated as a standard conjecture that every nef line bundle is semiample, and it is intimately related to the abundance conjecture. In this talk I will report on a recent progress on this problem in a joint work with Thomas Peternell.

Eyal Markman

Constructing algebraic cycles on products of K3 surfaces via hyperholomorphic bundles

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Let Z be a rational cohomology class of Hodge type $(2, 2)$ on the product $X \times Y$ of two K3 surfaces. If Z induces an isometry of the second rational cohomologies of X and Y , then Z is algebraic, by a recent result of Buskin, extending work of Mukai. We will report on Buskin's result and on related work in progress.

Shigeru Mukai

Root systems and automorphism groups of Enriques surfaces

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Enriques surfaces with only finitely many automorphisms were classified by Nikulin and Kondō in 80's. They are characterized by the property that every genus 1 fibration has Mordell-Weil rank 0. As a next case of this result, I classify Enriques surfaces which have a unique genus 1 fibration of positive Mordell-Weil rank modulo automorphisms, using their root systems.

Viacheslav V. Nikulin

Degenerations of Kahlerian K3 surfaces with finite symplectic automorphism groups

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Such degenerations of codimension 1 were classified in my recent preprints and papers. Here I shall consider classification of such degenerations of codimension greater than 1 for some groups

Thomas Peternell

Nef line bundles on Calabi-Yau varieties

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I will discuss nef line bundles L on Calabi-Yau manifolds, in particular in dimension 3. The basic problem is to show that L is semiample, i.e. , some multiple is generated by global sections. I will report on joint recent work with V.Lazic and K.Oguiso on this abundance type problem.

Kristian Ranestad

Kummer surfaces and hyperkähler fourfolds, as degeneracy loci

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Kummer quartic surfaces may be constructed as both symmetric and Lagrangian degeneracy loci. I shall discuss these constructions, relations between them, and similar constructions of hyperkähler fourfolds, in a report on common work with Atanas Iliev, Grzegorz and Michal Kapustka.

Alessandra Sarti

On the moduli space of smooth cubic threefolds and irreducible holomorphic symplectic manifolds

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In a famous paper Allcock, Carlson and Toledo describe the moduli space of smooth cubic threefolds as a ball quotient. Here we give an interpretation of this moduli space as moduli space of some special irreducible holomorphic symplectic fourfold with a non-symplectic automorphism of order three. This is part of a more general construction, that I will explain in the talk.

It is a joint work with S. Boissière and C. Camere.

Matthias Schütt

Gorenstein \mathbb{Q}_l -cohomology fake projective planes

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Inspired by fake projective planes, this talk concerns normal projective surfaces in zero or odd characteristic whose étale cohomology equals that of \mathbb{P}^2 . I shall concentrate on the case of rational double point singularities and numerically trivial canonical bundle. This setting which has been initiated over the complex numbers by Hwang-Keum-Hashi is closely related to Enriques surfaces. In my talk I will discuss the classification and lay out the subtleties and novelties featuring in odd characteristic. Time permitting, I will also discuss partial results in characteristic two.

Sho Tanimoto

On the geometry of thin exceptional sets in Manin's conjecture

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Manin's conjecture predicts asymptotic formulae for the counting functions of rational points on Fano varieties, and it has an explicit asymptotic formula in terms of geometric invariants of the underlying variety. The original conjecture which predicts an asymptotic formula after removing a proper closed subset is wrong due to the existence of covering families of subvarieties violating compatibility of Manin's conjecture, and its refinement, suggested by Peyre, removes thin sets instead of closed sets. In this talk, I would like to present some positive evidences of this conjecture using birational geometry, e.g., the minimal model program and the boundedness of log Fano varieties. This is joint work with Brian Lehmann.

Andrzej Weber

Hirzebruch class of quotient varieties

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Various constructions leading to new examples of varieties with trivial canonical bundle are based on resolutions of quotient singularities. Analytically such singularities look like \mathbb{C}^n/G , where $G \subset SL(n, \mathbb{C})$. Locally they admit a \mathbb{C}^* action. We study characteristic classes of such singularities and their crepant resolutions. We concentrate on the Hirzebruch class, although we will also mention the elliptic genus of Borisov and Libgober. These classes live in the equivariant cohomology of \mathbb{C}^n/G which can be identified with the ring of power series. We find that the Hirzebruch class (under some identification) coincides with the Molien series given in terms of the representation of G on $\Lambda(\mathbb{C}^n)$.

This is a joint work with Maria Donten-Bury